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FLUCTUATION HYDRODYNAMICS OF THE BROWNIAN MOTION OF A PARTICLE IN A FIXED DISPERSED LAYER*

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The influence of the perturbation exerted by a grid of fixed spherical particles, randomly distributed in space, on the Brownian diffusion of particles suspended in the flow of a fluid which penetrates the grid is disucssed. The fixed particles affect the coefficient of diffusion that is transverse to the flow in two ways: on the one hand they reduce it in accordance with the Stokes coefficient, and on the other they increase it because of the influence of a random velocity field which is generated by the flow past the randomly distributed particles. A convective diffusion equation is derived on the basis of the Fokker-Planck equation for a distribution function. A stochastic diffusion equation (of Langevin's type) obtained with a random velocity field is solved by the method of Green's function, whence the desired diffusion coefficient is found. The errors allowed when solving a similar problem in /1/ are indicated.

The fluctuation hydrodynamics of Brownian motion in a homogeneous viscous fluid was discussed in /2/ where, in particular, an expression for the coefficient of the particle resistance was obtained in terms of the fluctuation characteristics of the fluid. Later, the influence of hydrodynamic fluctuations on the diffusion of a particle in a homogeneous fluid was examined in /3/: it was shown that the diffusion coefficient of a particle that is large with respect to intermolecular distances is determined entirely by the thermal fluctuations of the fluid velocity field. This result was also confirmed by the microscope kinetic theory of Brownian motion in /4, 5/, where an expression similar to Kubo's formula, for the coefficient of resistance of a large particle in terms of the fluctuation

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of the stress tensor of the fluid was obtained, and it was shown that for an incompressible fluid the well-known Stokes formula follows from this expression.

In the present paper we consider the influence of fluctuation hydrodynamics on the diffusion of a suspended particle. The hydrodynamic fluctuations are generated by a fluid flow with velcoity $\mathbf{v}_0$ through a rarefied system of randomly distributed fixed particles. These fluctuations are superimposed on the thermal fluctuations of the fluid velocity, and result in an increase in the diffusion coefficient of a suspended particle. The purpose of the present paper is to analyse this effect.

1. The diffusion equation of a Brownian particle. In deriving the diffusion equation we shall proceed from the kinetic Fokker-Planck equation obtained in /6/ for an N-particle distribution function, $f^N(\{\mathbf{R}_i, \mathbf{P}_i\}, t)$, of the coordinates and momenta of the particles with mass M_i, suspended in a viscous fluid:

$$\frac{\partial f^N}{\partial t} + \sum\left[\frac{\mathbf{P}_i}{M_i}\frac{\partial f^N}{\partial \mathbf{R}_i} + (\mathbf{F}_i^h + \mathbf{F}_i^A)\frac{\partial f^N}{\partial \mathbf{P}_i}\right] = \sum_{ij}\frac{\partial}{\partial \mathbf{P}_i}\xi_{ij}\left[\left(\frac{\mathbf{P}_j}{M_j} - \mathbf{v}_{0j}\right)f^N - kT\frac{\partial f^N}{\partial \mathbf{P}_j}\right] \tag{1.1}$$

where $\mathbf{F}_i^h$ is the elastic force of the potential interaction (of the hard-sphere type) between the particles, $\mathbf{F}_i^A = {}^4/_3\pi a_i^3\nabla p_0$ is the Archimedean force acting on a particle of radius a_i in the fluid with pressure gradient ∇p_0, $\mathbf{v}_{0j}$ is the velocity of the fluid, not perturbed by the particles, at the point $\mathbf{R}_j$, ξ_{ii} is the resistance coefficient of the i-th particle, and ξ_{ij} $(i \neq j)$ are coefficients describing the hydrodynamic interaction between the particles: these coefficients are identical with those in the expression for the force acting on the particle in a system where the particles move in a viscous fluid with velocities $\mathbf{V}_i = \mathbf{P}_i/M_i$.

$$\mathbf{F}_i = -\sum_j \xi_{ij}(\mathbf{V}_j - \mathbf{v}_{0j}) \tag{1.2}$$

In the case under consideration, the positions of all particles with the exception of one $(i = 1)$ are fixed, that is $\mathbf{V}_i = \mathbf{V}_1\delta_{1i}$, where δ_{ij} is the Kronecker delta. For simplicity we shall assume that the velocity $\mathbf{v}_0$ of non-perturbed flow does not depend on the coordinates, and is directed along the z-axis.

We obtain the diffusion equation of a Brownian particle as the first moment equation (1.1) by integrating the right and left sides with respect to the coordinates of all fixed particles and to the momenta of all particles. This gives

$$\frac{\partial}{\partial t}c - \frac{\partial}{\partial \mathbf{R}_1}c\mathbf{v}_0 - \frac{\partial}{\partial \mathbf{R}_1}c\mathbf{V}_d = 0 \tag{1.3}$$

$$c(\mathbf{R}_1, t) = \langle 1\rangle, \quad c(\mathbf{R}_1, t)\mathbf{V}_d(\mathbf{R}_1, t) = \langle \mathbf{P}_1/M - \mathbf{v}_0\rangle$$

$$\langle A\rangle \equiv \int d\mathbf{R}_2 \ldots d\mathbf{R}_N\, d\mathbf{P}_1 \ldots d\mathbf{P}_N A f^N$$

The second term on the left of this equation describes the convective transfer of the particles, and the thrid the diffusion transfer. Since the diffusion velocity $\mathbf{V}_d$ of the Brownian particle is unknown, the equation obtained is not closed. To close it, we shall use the second moment equation which can be obtained from (1.1) by mutliplying all its terms by $(\mathbf{P}_1 - M_1\mathbf{v}_0)$, and performing the subsequent integration with respect to $\mathbf{R}_2, \ldots, \mathbf{R}_N, \mathbf{P}_1, \ldots, \mathbf{P}_N$. In the approximation which is stationary with respect to the diffusion velocity, using for (1.3) this equation reduces to the form

$$\frac{\partial}{\partial \mathbf{R}_1}ckT - (\overline{\mathbf{F}_1^h} + \mathbf{F}_1^A)c = -c\mathbf{V}_d\bar{\xi}_{11} + c\sum_{j\neq 1}\bar{\xi}_{1j}\mathbf{v}_0 \tag{1.4}$$

where the following moments are used:

$$\langle(\mathbf{P}_1 - M_1\mathbf{V}_d - M_1\mathbf{v}_0)(\mathbf{P}_i - M_i\mathbf{V}_d - M_i\mathbf{v}_0)\rangle = \delta_{1i}M_1c(\mathbf{R}_1, t)kT$$

$$\left\langle(\mathbf{F}_i^h + \mathbf{F}_i^A)\mathbf{P}_1\frac{\partial}{\partial \mathbf{P}_i}\right\rangle = -\delta_{1i}c(\mathbf{R}_1, t)(\overline{\mathbf{F}_1^h} + \mathbf{F}_1^A)$$

$$\left\langle\mathbf{P}_1\frac{\partial}{\partial \mathbf{P}_i}\xi_{ii}\left(\frac{\mathbf{P}_i}{M_i} - \mathbf{v}_0 + kT\frac{\partial}{\partial \mathbf{P}_i}\right)\right\rangle = -\delta_{1i}c(\mathbf{R}_1, t)\mathbf{V}_d\bar{\xi}_{11}$$

$$\left\langle\mathbf{P}_1\frac{\partial}{\partial \mathbf{P}_i}\sum_{j(\neq i)}\xi_{ij}\left(\frac{\mathbf{P}_j}{M_j} - \mathbf{v}_0 + kT\frac{\partial}{\partial \mathbf{P}_j}\right)\right\rangle = \delta_{1i}\mathbf{v}_0c(\mathbf{R}_1, t)\sum_{j(\neq 1)}\bar{\xi}_{1j}$$

The bars indicate averaging over the coordinates of the fixed particles.

We shall perform all further calculations in the point-particle approximation. In this hydrodynamic approximation the sizes of the particles are considered to be much smaller than any other intrinsic parts of the system, including the mean distances between the particles. As we know, such an approximation produces an error of the order of the volume concentration of the φ-particles, and therefore we can ignore the terms with $\bar{\mathbf{F}}^h$ and $\mathbf{F}^A$ which are proportional to φ. Then the expression for diffusion flow,

$$J \equiv c\mathbf{V}_d = -\bar{\xi}_{11}^{-1} kT\nabla c + \bar{\xi}_{11}^{-1} \sum_{j\neq 1} \bar{\xi}_{1j}\mathbf{v}_0 c$$

follows from (1.4).

On substituting it into the first momentum equation (1.3) we obtain the equation of diffusion

$$\partial c/\partial t + (\mathbf{v}_0 + \bar{\mathbf{v}})\nabla c - D\nabla^2 c = 0 \tag{1.5}$$

$$D = kT\bar{\xi}_{11}^{-1}, \quad \bar{\mathbf{v}} = \bar{\xi}_{11}^{-1} \sum_{j\neq 1} \bar{\xi}_{1j}\mathbf{v}_0 \tag{1.6}$$

where D is the diffusion coeffitient, and $\bar{\mathbf{v}}$ is the mean perturbation of the stream velocity, dependent on the flow around the random configuration of fixed particles.

2. The stochastic diffusion equation. The subsequent analysis will be similar to that carried out in /3/, where it was shown that the diffusion of one large particle in a pure fluid is entirely defined by the fluid-velocity thermal fluctuations, and the initial diffusion coefficient which does not take into account the influence of these fluctuations was assumed to be zero.

In the system under consideration there occur not only thermal fluctuations of the fluid velocity but also random perturbations arising in the flow around fixed spheres. The influence of the thermal fluctuations has already been allowed for in expression (1.6) for D (this was done in /6/ in deriving the Fokker-Planck equation (1.1)). As shown above, the method of deriving the convective diffusion equation from the Fokker-Planck equation includes averaging of the perturbation of the convective flow over the coordinates of the randomly distributed fixed particles. Allowing for the fluctuation of this perturbation leads to an additional modification of the diffusion coefficient.

Consider, instead of the diffusion equation (1.5), the corresponding Langevin stochastic equation in which not the mean disturbance of the flow velocity but the random quantity $\mathbf{v}(\mathbf{r}) = \bar{\xi}_{11}^{-1}\Sigma\xi_{1j}\mathbf{v}_0$ occurs. Introducing a delta-type source into the right side of this equation we can write it immediately for Green's function or for a propagator $G(\mathbf{r}, \mathbf{r}_0, t, t_0)$ which describes the probability of detecting the particle at a point $\mathbf{r}$ at the instant t under the condition of finding it at $\mathbf{r}_0$ at the initial instant t_0.

$$(\partial/\partial t - D\nabla^2 + (\mathbf{v}_0 + \mathbf{v}(\mathbf{r}))\cdot\nabla)\, G(\mathbf{r}, \mathbf{r}_0, t, t_0) = \delta(\mathbf{r}-\mathbf{r}_0)\,\delta(t-t_0) \tag{2.1}$$

with the boundary conditions $G(\mathbf{r}, \mathbf{r}_0, t, t_0) = 0$ for $t < t_0$ and $(\nabla G)\cdot\mathbf{n} = 0$ on the surface of each particle ($\mathbf{n}$ is the normal to this surface). By the definition of a propagator we have

$$\langle(\mathbf{r}-\mathbf{r}_0)^n\rangle_d = \int d\mathbf{r}\,(\mathbf{r}-\mathbf{r}_0)^n\, G(\mathbf{r}, \mathbf{r}_0, t, t_0) \tag{2.2}$$

hence, allowing from (2.1) in the approximation of point particles, i.e. ignoring the surface integrals we obtain

$$\frac{\partial}{\partial t}\langle(\mathbf{r}-\mathbf{r}_0)\rangle_d = \mathbf{v}_0 + \langle\mathbf{v}(\mathbf{r})\rangle_d \tag{2.3}$$

$$\frac{\partial}{\partial t}\langle(r_\alpha - r_{0\alpha})^2\rangle_d = 2D\Theta(t) + 2\langle(r_\alpha - r_{0\alpha})(v_{0\alpha} + v_\alpha(\mathbf{r}))\rangle_d \tag{2.4}$$

For computing the right sides of these equations we must find the explicit form of the propagator $G(\mathbf{r}, \mathbf{r}_0, t, t_0)$. From the Fourier transform of Eq.(2.1),

$$(-i\omega - D\nabla^2 + \mathbf{v}_0\cdot\nabla)\, G(\mathbf{r}, \mathbf{r}_0, \omega) = \delta(\mathbf{r}-\mathbf{r}_0) - \mathbf{v}(\mathbf{r})\cdot\nabla G(\mathbf{r}, \mathbf{r}_0, \omega)$$

there follows the integral equation

$$G(\mathbf{r}, \mathbf{r}_0, \omega) = G_0(\mathbf{r}, \mathbf{r}_0, \omega) - \int d\mathbf{r}' G_0(\mathbf{r}, \mathbf{r}', \omega)\,\mathbf{v}(\mathbf{r}')\cdot\nabla_{\mathbf{r}'} G(\mathbf{r}', \mathbf{r}_0, \omega)$$

where $G_0(\mathbf{r}, \mathbf{r}_0, \omega)$ is Green's function of the equation

$$(-i\omega - D\nabla^2 + \mathbf{v}_0\cdot\nabla)\, G_0 = \delta(\mathbf{r}-\mathbf{r}_0)$$

We find by an iteration method the solution of the integral equation $G(\mathbf{r}, \mathbf{r}_0, \omega)$ in the form of a series in powers of the perturbation of velocity $\mathbf{v}(\mathbf{r})$. To a first approximation it has the form

$$G(\mathbf{r}, \mathbf{r}_0, \omega) = G_0(\mathbf{r}, \mathbf{r}_0, \omega) - \int d\mathbf{r}' G_0(\mathbf{r}, \mathbf{r}', \omega)\,\mathbf{v}(\mathbf{r}')\cdot\nabla_{\mathbf{r}'} G_0(\mathbf{r}', \mathbf{r}_0, \omega)$$

On subsituting this relation into (2.4), for the root-mean-square shift in a direction transverse to $\mathbf{v}_0$ we obtain

$$-i\omega\overline{\langle(x-x_0)^2\rangle_d}=\frac{2Di}{\omega}+2\int d\mathbf{r}\,(x-x_0)\overline{v_x(\mathbf{r})}\,G_0(\mathbf{r}-\mathbf{r}_0,\omega)-2\int d\mathbf{r}\,d\mathbf{r}'\,(x-x_0)\,G_0(\mathbf{r}-\mathbf{r}',\omega)\,\overline{v_x(\mathbf{r})\,v_\alpha(\mathbf{r}')}\,\frac{\partial}{\partial r'_\alpha}G_0(\mathbf{r}'-\mathbf{r}_0,\omega)\tag{2.5}$$

Introducing into (2.5) the new variables $\rho=\mathbf{r}-\mathbf{r}_0$, $\rho'=\mathbf{r}'-\mathbf{r}_0$ and integrating with respect to each of them over the whole volume (which gives an error of the order of φ), taking into account the fact that $\overline{v_x(\mathbf{r})}=0$) we find

$$-i\omega\overline{\langle(x-x_0)^2\rangle_d}=\frac{2Di}{\omega}-2\int d\rho\,d\rho'\rho_x G_0(\rho-\rho',\omega)\,\Gamma_{x\alpha}(\rho-\rho')\frac{\partial}{\partial\rho_\alpha}G_0(\rho',\omega)$$

$$(\Gamma_{x\alpha}(\mathbf{r}-\mathbf{r}')=\overline{v_x(\mathbf{r})\,v_\alpha(\mathbf{r}')}\,)$$

The integral term is a convolution; it is therefore convenient to write it in the spatial Fourier form

$$-i\omega\overline{\langle(x-x_0)^2\rangle_d}=\frac{2Di}{\omega}+\frac{2i}{\omega(2\pi)^6}\int d\mathbf{k}\Gamma_{xx}(-\mathbf{k})\,G_0(\mathbf{k},\omega)\tag{2.6}$$

$$G_0(\mathbf{k},\omega)=(-i\omega+Dk^2+iv_0k_z)^{-1}\tag{2.7}$$

where the relation $\rho_x(\mathbf{k})=-i(2\pi)^3\partial\delta(\mathbf{k})/\partial k_x$ is used.

Thus, we can see from (2.6) that in the limit as $\omega\to0$, the effective diffusion coefficient in the transverse direction is

$$D_\perp=D+\Delta D_\perp=D+\frac{1}{(2\pi)^3}\int d\mathbf{k}\Gamma_{xx}(-\mathbf{k})\,G_0(\mathbf{k},0)\tag{2.8}$$

The explicit form of all quantities in this expression and its final form are obtained below.

3. Calculation of the coefficient ξ_{11} and the perturbations of the fluid velocity. Consider a particle which moves with velocity $\mathbf{V}_i=\delta_{i1}\mathbf{V}_1$ in a fluid whose velocity when this particle is not present is $\mathbf{v}_0+\mathbf{v}_i$. The particle will be subjected to the resistance force

$$F_i=-\zeta_i(\mathbf{V}_i-\mathbf{v}_0-\mathbf{v}_i);\quad \zeta_i=6\pi\eta a_i,\ a_i=\delta_{1i}a_1+(1-\delta_{1i})\,a_2\tag{3.1}$$

where ζ_i is the Stokes coefficient of resistance of the particle with radius a_i. In turn, this particle generates the perturbation of the fluid flow, which in the approximation of point particles can be written as

$$\Delta\mathbf{v}(\mathbf{r})=T(\mathbf{r}-\mathbf{R}_i)\cdot(-\mathbf{F}_i)=\zeta_iT(\mathbf{r}-\mathbf{R}_i)\cdot(\mathbf{V}_i-\mathbf{v}_0-\mathbf{v}_i)$$

$$T(\mathbf{r})=(8\pi\eta r)^{-1}(U+\mathbf{rr}/r^2)$$

where $T(\mathbf{r})$ is the Oseen tensor which is Green's function of Stokes equation, and U is the unit tensor. Hence we have

$$\mathbf{v}_i=-\sum_j\zeta_jT_{ij}\cdot(\mathbf{v}_0-\mathbf{V}_j-\mathbf{v}_j),\quad T_{ij}=T(\mathbf{R}_i-\mathbf{R}_j)$$

and by one of the iteration methods we obtain

$$\mathbf{v}_i=-\sum_j\zeta_jT_{ij}\cdot(\mathbf{v}_0-\mathbf{V}_j)-\sum_{jk}\zeta_j\zeta_kT_{ij}T_{jk}\cdot(\mathbf{v}_0-\mathbf{V}_k)-\sum_{jkl}\zeta_j\zeta_k\zeta_lT_{ij}T_{jk}T_{kl}\cdot(\mathbf{v}_0-\mathbf{V}_l)-\ldots\tag{3.2}$$

By (1.2) and (3.1) we have the following expression for the force acting on a particle:

$$\sum_j\xi_{ij}(\mathbf{V}_j-\mathbf{v}_0)=\zeta_i(\mathbf{V}_i-\mathbf{v}_0-\mathbf{v}_i)\tag{3.3}$$

On substituting into it the expansion (3.2) for $\mathbf{v}_i$, and equating the coefficients of $\mathbf{V}_i-\mathbf{v}_0$, we find

$$\xi_{ii}=\zeta_i+\zeta_i^2\sum_j\zeta_jT_{ij}T_{ji}-\zeta_i^2\sum_{jk}\zeta_j\zeta_kT_{ij}T_{jk}T_{ki}+\ldots$$

Averaging all the terms of this series over the positions of each of the intermediate particles (over which the summation is carried out), we obtain

$$\overline{\xi}_{11}=\zeta_1+\zeta_1^2\zeta_2c_2\int d\mathbf{R}_2T_{12}T_{21}-\zeta_1^2\zeta_2^2c_2^2\int d\mathbf{R}_2\,d\mathbf{R}_3T_{12}T_{23}T_{31}+\ldots,\quad c_2=N_2/\Omega$$

where N_2 is the number of fixed particles in the system, and Ω is the system's volume. In the Fourier representation, where $T(\mathbf{k}) = (U - \mathbf{kk}/k^2)/(\eta k^2)$, using the convolution theorem we express this series in the form

$$\bar{\xi}_{11} = \zeta_1 + \zeta_1^2\zeta_2 c_2 \int \frac{d\mathbf{k}}{(2\pi)^3}\left(U - \frac{\mathbf{kk}}{k^2}\right)\left[\left(\frac{1}{\eta k^2}\right)^2 - \zeta_2 c_2\left(\frac{1}{\eta k^2}\right)^3 + \zeta_2^2 c_2^2\left(\frac{1}{\eta k^2}\right)^4 - \dots\right] = \zeta_1 + \zeta_1^2\zeta_2 c_2 \int \frac{d\mathbf{k}}{(2\pi)^3}\left(U - \frac{\mathbf{kk}}{k^2}\right)\frac{1}{\eta k^2 + \zeta_2 c_2}$$

(here we use the expression in square brackets for the sum of a geometric progression). After evaluating the integral we find that $\bar{\xi}_{11}$ is a diagonal tensor with the coefficients

$$\bar{\xi}_{11} = \zeta_1\left(1 + 3\frac{a_1}{a_2}\sqrt{\frac{\varphi_2}{2}}\right); \quad \varphi_2 = \frac{4}{3}\pi a_2^3 c_2 \tag{3.4}$$

where φ_2 is the volume density of the fixed particles.

Averaging (3.2) over the coordinates of all the fixed particles we can satisfy ourselves that in the lower order with respect to φ_2 the mean perturbation of the flow velocity is in fact determined by the expression in Eq.(1.5). Therefore the correlation function $\Gamma_{xx}(\mathbf{k})$ in (2.6) can be found in terms of the series (3.2).

Let us write the averaged product of this series

$$\overline{\mathbf{v}(\mathbf{r})\mathbf{v}(\mathbf{r}')} = \{[-\zeta_2\sum_i T_{ri} + \zeta_2^2\sum_{ij} T_{rj}T_{ji} + \zeta_2^3\sum_{ijk} T_{rk}T_{kj}T_{ji} + \dots] \times [-\zeta_2\sum_l T_{r'l} + \zeta_2^2\sum_{lm} T_{r'm}T_{ml} - \zeta_2^3\sum_{lmn} T_{r'n}T_{nm}T_{ml} + \dots]\}_{av} : \mathbf{v}_0\mathbf{v}_0 \tag{3.5}$$

where the braces $\{\Phi\}_{av}$ are equivalent to the bar over Φ. By multiplying the square brackets, we can represent (3.5) in the form

$$\overline{\mathbf{v}(\mathbf{r})\mathbf{v}(\mathbf{r}')} = (\lambda_1 - \lambda_2 + \lambda_3 - \dots) : \mathbf{v}_0\mathbf{v}_0 \tag{3.6}$$

$$\lambda_1 = \frac{\zeta_2^2}{\Omega}\sum_i \int d\mathbf{R}_i T_{ri}\left[T_{r'i} - \frac{\zeta_2}{\Omega}\sum_m \int d\mathbf{R}_m T_{r'm}T_{mi} - \dots\right]$$

$$\lambda_2 = \frac{\zeta_2^2}{\Omega^2}\sum_{\substack{il\\(l\neq i)}} \int d\mathbf{R}_i\, d\mathbf{R}_l T_{ri}\left[T_{r'l} - \frac{\zeta_2}{\Omega}\sum_m \int d\mathbf{R}_m T_{r'm}T_{ml} - \dots\right]$$

$$\lambda_3 = \frac{\zeta_2^3}{\Omega^2}\sum_{ij} \int d\mathbf{R}_i\, d\mathbf{R}_j T_{rj}T_{ji}\left[T_{r'i} - \zeta_2 T_{r'j}T_{ji} - \frac{\zeta_2}{\Omega}\sum_{m(\neq j)} \int d\mathbf{R}_m T_{r'm}T_{mi} - \dots\right]$$

The expression for λ_1 corresponds to the product of the first term from the first square brackets in (3.5) by all terms from the second square brackets with $l = i$, and is of the order of c, and the expression for λ_2 with $l \neq i$ is of the order of c^2. Therefore, in the case of small c we can ignore the contribution of λ_2. Physically, this means that the basic contribution to the correlation function $\Gamma(\mathbf{r} - \mathbf{r}')$ is that of the disturbances generated in the same i-th particle (with subsequent summation over all i).

Let us now look into λ_3. The second term contains the factor T_{ji} identical with that in front of the square bracket. Both of them describe the propagation of the perturbation from the i-th to the j-th particle. Such a double description of the same process is physically unjustified, therefore the term with T_{ij} can be simply dropped since it has a higher order of smallness with respect to a_2/R_{ij} compared with the first term in λ_3. For the same reason, in all subsequent λ_n $(n > 3)$ we can omit the terms which are the products from different square brackets with identical indices in (3.5). Then (3.6) takes the form

$$\overline{\mathbf{v}(\mathbf{r})\mathbf{v}(\mathbf{r}')} = \zeta_2^2 c_2 \int d\mathbf{R}_2 [T_{r2} - \zeta_2 c_2 \int d\mathbf{R}_3 T_{r3}T_{32} + \zeta_2^2 c_2^2 \int d\mathbf{R}_3\, d\mathbf{R}_4 T_{r4}T_{43}T_{32} - \dots][T_{r'2} - \zeta_2 c_2 \int d\mathbf{R}_3 T_{r'3}T_{32} + \zeta_2^2 c_2^2 \int d\mathbf{R}_3\, d\mathbf{R}_4 T_{r'4}T_{43}T_{32} - \dots] : \mathbf{v}_0\mathbf{v}_0 \tag{3.7}$$

or, in the Fourier representation,

$$\Gamma_{xx}(\mathbf{k}) = \zeta_2^2 c_2 v_0^2 [T(\mathbf{k}) - \zeta_2 c_2 (T(\mathbf{k}))^2 - \zeta_2^2 c_2^2 (T(\mathbf{k}))^3 - \dots]_{xz}^2 = \zeta_2^2 c_2 v_0^2 \left(\frac{1}{\eta k^2}\right)^2 \frac{k_x^2 k_z^2}{k^4(1 + \zeta_2 c_2/(\eta k^2))^2} = (6\pi a_2 v_0)^2 c_2 \frac{k_x^2 k_z^2}{k^4(k^2 + \varkappa^2)^2} \tag{3.8}$$

$$(\varkappa = \sqrt{\zeta_2 c_2/\eta} = \sqrt{6\pi a_2 c_2})$$

where $\varkappa$ is the inverse length of the screening in a dispersion system (in the approximation of point particles).

4. The diffusion coefficient. On substituting expression (2.7) into (2.8) and (3.8), we obtain

$$\Delta D_\perp = -\frac{(6\pi a_2 v_0)^2 c_2}{(2\pi)^3}\int d\mathbf{k}\frac{k_x{}^2 k_z{}^2}{k^4(k^2+\varkappa^2)^2(Dk^2+iv_0k_z)} \tag{4.1}$$

On evaluating this integral we find that

$$\Delta D_\perp = \frac{9}{2}\pi a_2{}^2 D c_2 \varkappa^{-1}\left[\frac{1}{3}+\alpha(1-2\alpha)-\frac{\alpha}{2}(1-4\alpha^2)\ln\frac{\alpha+1}{\alpha}\right],$$
$$\alpha = \frac{\varkappa D}{v_0}$$

In the limiting case $\alpha \ll 1$, this expression takes the form

$$\Delta D_\perp = \frac{3}{2}\pi a_2{}^2 D c_2 \varkappa^{-1} = \frac{3}{4}\sqrt{\frac{\varphi_2}{2}}D \tag{4.2}$$

It follows from (2.8) and (4.2) that

$$D_\perp = D\left(1+\frac{3}{4}\sqrt{\frac{\varphi_2}{2}}\right) \tag{4.3}$$

The coefficient D also depends on the concentration of the fixed particles. In fact, by the definition (1.6) of D and expression (3.4),

$$D = D_0\left(1-3\frac{a_1}{a_2}\sqrt{\frac{\varphi_2}{2}}\right), \quad D_0 = \frac{kT}{6\pi a_1\eta} \tag{4.4}$$

where D_0 is the diffusion coefficient of a particle of radius a_1 in a pure fluid with viscosity η. Thus finally, from (4.3) and (4.4) we obtain

$$D_\perp = D_0\left[1-3\left(\frac{a_1}{a_2}-\frac{1}{4}\right)\sqrt{\frac{\varphi_2}{2}}\right] \tag{4.5}$$

Hence it is seen that, depending on the ratio a_1/a_2, the diffusion coefficient $D_\perp$ can be larger or smaller than the diffusion coefficient D_0 in a pure fluid. This is connected with the change in the relative contribution of two competing influences of the fixed admixtures on the diffusion of a Brownian particle. On the one hand, the presence of immobile admixtures increases the coefficient of the resistance to the motion of particle ξ_{11}, and correspondingly reduces the diffusion coefficient (4.4). On the other hand, in a flow past a random configuration of the admixtures there appears a random velocity field which leads to an additional fluctuation motion (relatively to the thermal motion) of the particle and, correspondingly, to an increase in the diffusion coefficient.

The calculation of the coefficient $D_\perp$ in the system described was also discussed in /1/. However, the result obtained there is not exactly true since the authors did not take into account the difference between D and D_0, given by (4.4). Also, in computing $\Delta D_\perp$ they used incorrect expression for $\Gamma_{xx}(\mathbf{k})$ in the form of a product $c_2u_x(\mathbf{k})u_x(-\mathbf{k})$ where $\mathbf{u}(\mathbf{k})$ is the Fourier transform of a non-screened velocity field of one mobile particle. Such a choice of $\Gamma_{xx}(\mathbf{k})$ corresponds to allowing only for the first term of λ_1 in expansion (3.6), or the first term in the first square brackets in (3.7). In computing $\Delta D_\perp$ this leads to a double result compared with (4.2). Besides this the authors made a mistake in evaluating the integral for $\Delta D_\perp$, and their correction of the transverse-diffusion coefficient calculated was four times as great as that given by (4.2) in the present paper.

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